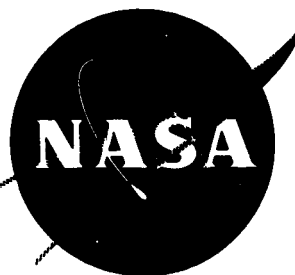


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by William H. Roudebush and David A. Mandell
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INTRODUCTION

Many factors influence the amount of gas required to pressurize a cryogenic propellant tank during the period of outflow. Besides the tank volume and the temperature and pressure of the incoming gas, other factors such as outlet flow rate, gas-to-wall heat-transfer coefficient, mass and specific heat of the tank wall, and the gas specific heat must be considered. A systematic experimental investigation of these individual factors is very difficult for liquid hydrogen. It is desirable, therefore, to attempt analytically to determine the relative significance of the various parameters.

An analysis of the tank pressurization problem for a cylindrical tank was made at Lewis Research Center (ref. 1). A simple one-dimensional model was used, based on a rather restrictive set of physical assumptions. Even for the simple model the resulting differential equations were quite complex and a numerical solution was clearly indicated. The details of the numerical solution were worked out and a computer program was developed. Results of the analysis were compared with experimental results for a number of cases and the agreement was shown to be surprisingly good in view of the restrictive assumptions.

The good agreement appeared to justify the use of the computer program for investigating systematically the various parameters affecting the pressurization problem. This investigation was carried out and the results are presented in detail in a forthcoming report (ref. 2). A brief discussion of these results and of the assumptions involved is given in the present paper.

ANALYSIS

The analysis is restricted to the cylindrical portion of the tank (fig. 1) and only the period of time during which outflow occurs is considered. Certain assumptions are made in an attempt to simplify the analysis and shorten the subsequent numerical solution while still retaining the most important features of the problem. A list of the assumptions and a discussion of their validity follows:

- (1) The ullage gas is nonviscous.
- (2) The velocity of the ullage gas is parallel to the tank axis and varies only in the axial direction.

- (3) The tank pressure varies only with time.
- (4) The ullage gas temperature varies only in the axial direction.
- (5) The tank wall temperature varies only in the axial direction.
- (6) No heat is transferred axially in either the gas or the wall.
- (7) No condensation or evaporation occurs.
- (8) No heat is transferred at the liquid interface or at the top of

the tank.

With these assumptions the problem is reduced to a one-dimensional, nonsteady, nonviscous flow of the ullage gas with heat transfer to the tank wall.

Although the problem is clearly not one-dimensional (radial flow must take place as the gas enters the tank), it is necessary to simplify the equations. Therefore, assumptions (1) and (2) stipulate that the pressurizing gas enters the tank uniformly at $x = 0$ (fig. 1) and proceeds downward with a velocity that varies with time and axial location only; that is, no mixing of the ullage gas occurs.

Assumption (3) is likely to be satisfied closely because of the low gas density and small change in gas momentum from top to bottom of the tank.

Assumption (4) arises from experimental results obtained at Lewis with a cylindrical tank having a low heat leak. The assumption may not be valid for other circumstances.

Assumption (5) is adequate for thin metal tank walls.

Assumption (6) arises from the low conductivity of the ullage gas and the small thickness of the tank wall.

Assumption (7) appeared to be justified by early data taken at Lewis. Recently taken data, however, put the assumption in doubt. More experimental results, especially on larger tanks, are needed to evaluate this assumption properly.

Assumption (8) has not been verified. There are likely to be some cases in which the heat transfer to the top of the tank, at least, cannot be ignored.

With these assumptions, the differential equations that govern the pressurization problem can be written (see ref. 1 for details)

$$\frac{\partial T}{\partial t} = \frac{2hZRT}{rMPc_p} (T_w - T) - u \frac{\partial T}{\partial x} + \frac{RT}{MJPC_p} \left(Z + T \frac{\partial Z}{\partial T} \right) \frac{\partial P}{\partial t} + \frac{RTZCq_I}{\pi r^2 MPc_p} \quad (1)$$

$$\frac{\partial T_w}{\partial t} = \frac{h}{l_w \rho_w c_w} (T - T_w) + \frac{q_o}{l_w \rho_w c_w} \quad (2)$$

$$\frac{\partial u}{\partial x} = \frac{1}{ZT} \left(Z + T \frac{\partial Z}{\partial T} \right) \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) - \frac{1}{ZP} \left(Z - P \frac{\partial Z}{\partial P} \right) \frac{\partial P}{\partial t} \quad (3)$$

(All symbols are defined in the appendix.) In addition to these three differential equations in the three unknowns T , T_w , and u , the following initial and boundary conditions are also required to determine a solution:

(1) At the start of outflow, the gas and wall temperature distributions must be given.

(2) The variation during outflow of the incoming gas temperature, the tank pressure, the outlet flow rate, and the gas and wall temperatures at the interface must be prescribed.

Furthermore, the heat-transfer coefficient must be supplied, either by an equation relating it to fluid properties or by using appropriate experimental values.

NUMERICAL SOLUTION

A finite difference solution of equations (1) to (3) was programmed in Fortran IV for use on an IBM 7094-II computer. Backward difference equations were used resulting in a nonlinear set of algebraic equations that were explicit in the unknown variables.

The time step Δt is related to the space step Δx by the requirement that

$$\Delta t = \frac{\Delta x}{u_L(t)}$$

where $u_L(t)$ is the velocity of the liquid surface. This restriction on Δt is used to keep the net spacing Δx constant as the solution progresses. (It is not a condition for stability of the numerical solution and it does not result in unusually small values of Δt). The program has been run over a very wide range of problems and no numerical instability has been encountered.

The output of the computer program is the distribution of gas and wall temperatures at any desired time during outflow. The pressurant mass required at each instant is also determined. A typical solution uses about 200 net points in the x -direction for covering the entire length of the tank. The 19 solutions presented in reference 1 averaged 24 seconds of computer time per solution.

EXPERIMENTAL AND CALCULATED RESULTS

In reference 3 the authors report some of the results of a systematic series of liquid hydrogen expulsion experiments. The tank used was 27 inches in diameter and 89 inches in overall length with dished head ends. A gas diffuser was used at the inlet. The tank was constructed of 5/16-inch 304 stainless-steel plate and was vacuum jacketed. The instrumentation, described in detail in reference 3, provided a relatively significant heat sink in some of the experiments.

Ten experiments (some of which were not discussed in ref. 3) were selected to check the analysis. These covered a wide range of outlet flow rates, tank pressures, and inlet gas temperature variations. Helium was used to pressurize in four of the cases. The detailed input data necessary to carry out the calculation is given in reference 1 for each of the experiments. Some of the principal data are given in table I.

TABLE I. - LEWIS EXPERIMENTAL DATA

Example	Pressure, lb/sq in.	Outflow time, sec	Outflow rate, cu ft/sec	Experimental average heat-transfer coefficient, Btu/(sq ft)(hr)(°R)	Pressur- izing gas
1	160	350	0.0669	13.75	H ₂
2	161	93	.2375	12.25	H ₂
3	57	284	.0780	7.09	H ₂
4	58	101	.2238	6.67	H ₂
5	164	95	.2340	11.34	H ₂
6	40	88	0.2550	5.13	H ₂
7	159	355	.0634	12.31	He
8	159	90	.2598	11.15	He
9	159	100	.2365	10.45	He
10	40	309	.0703	5.25	He

Figure 2 shows the gas and wall-temperature distributions calculated at the end of outflow and the corresponding experimental values for each example. The agreement generally is good.

Reference 4 reports the results of hydrogen experiments carried out at Lockheed-Georgia Company using a 40-inch-diameter test tank 100 inches in overall length. The test tank was 0.090-inch-thick stainless steel and was enclosed in a 60-inch-diameter vacuum-tight carbon steel tank. A gas diffuser was in the top and an antivortex baffle was in the bottom. Perforated conical slosh baffles were located at various axial positions. The heat sink effect of the internal hardware could not be well estimated from the information reported.

Nine tests are reported in reference 4 for which the system vacuum was maintained. These cover two values of inlet gas temperature and a range of values of

initial ullage. The outflow time and tank pressure varied only slightly from test to test. Helium was used to pressurize in one case. Sloshing of the liquid was induced in all but one case. The detailed input data for the calculations is given in reference 1. Some of the principal data are shown in table II.

TABLE II. - LOCKHEED-GEORGIA EXPERIMENTAL DATA

Example	Pressure, lb/sq in.	Outflow time, sec	Outflow rate, cu ft/sec	Experimental average heat-transfer coefficient, Btu/(sq ft)(hr)(°R)	Pressur- izing gas
1	45.5	89	^a 0.672	11.5	H ₂
2	47.6	103	.560	^b 12.0	H ₂
3	46.5	120	.511	11.3	H ₂
4	46.5	87	.607	12.0	H ₂
5	45.5	99	.609	12.1	He
6	47.0	95	.644	12.3	H ₂
7	45.0	111	.530	11.8	H ₂
8	46.2	97	.632	11.7	H ₂
9	45.5	105	.565	13.9	H ₂

^aFlow rates are computed from reported outflow time, tank volume, and percent initial ullage.

^bEstimated value; not given in reference 4.

For the Lewis and the Lockheed-Georgia experiments pressurant mass requirements were obtained from the analysis. Table III shows these calculated values along with the experimental value in each case. The percent difference is also shown. The average difference for the Lewis experiments is about 5 percent. The average difference for the Lockheed-Georgia experiments is about $4\frac{1}{2}$ percent. This agreement is better than might be expected from the simple description of the problem used for the analysis.

TABLE III. - PRESSURANT MASS REQUIREMENTS

Example	Experimental mass, lb	Calculated mass, lb	Percent differ- ence	Example	Experimental mass, lb	Calculated mass, lb	Percent differ- ence
Lewis data				Lockheed-Georgia data			
1	3.98	3.95	-0.75	1	2.61	2.81	7.67
2	2.72	2.60	-4.41	2	2.13	2.24	5.17
3	1.76	1.68	-4.54	3	2.86	3.05	6.64
4	1.24	1.27	2.42	4	2.57	2.65	3.11
5	3.76	3.51	-6.65	5	5.79	5.89	1.73
6	.83	.93	12.04	6	2.47	2.58	4.45
7	8.14	7.61	-6.51	7	2.81	2.86	1.78
8	5.59	5.57	-.36	8	2.81	2.95	4.98
9	9.24	8.48	-8.23	9	2.88	3.00	4.17
10	2.70	2.56	-5.18				

It should be noted that experimental average values of heat-transfer coefficient were used and that the gas and wall temperature distributions at the start of outflow were obtained from the data. The variation of inlet gas temperature with time at the position $x = 0$ is also from the experiments.

PARAMETRIC ANALYSIS

The agreement shown between calculated and experimental values of pressurant mass requirement in the preceding section encourages the use of the analytical method for examining the effect of the various parameters entering the pressurization problem. A method for doing this and the results obtained are described briefly in this section.

Dimensionless Parameters

The following additional assumptions are made to simplify the differential equations (1) to (3) and the initial and boundary conditions:

(9) The ullage gas is a perfect gas with constant specific heat.

(10) The gas-to-wall heat-transfer coefficient is constant in space and time for a given example.

(11) The inlet gas temperature, the tank pressure, and the outflow rate are constant.

(12) The gas and wall temperatures at the liquid interface are constant and equal throughout the outflow period.

(13) The gas and wall temperatures at the start of outflow are equal and vary linearly in the direction of the tank axis from the temperature at the liquid interface to a temperature at the top of the tank equal to the average of the inlet gas temperature and the liquid surface temperature.

The last assumption is obviously an arbitrary choice for the initial gas and wall temperatures. The effect of this assumption and the others will be considered later.

Using these assumptions and introducing the dimensionless variables

$$\hat{t} = \frac{t}{t_f}$$

$$\hat{x} = \frac{x}{L_f - L_o}$$

$$\hat{u} = \frac{u}{u_L}$$

$$\begin{aligned}\hat{T} &= \frac{T}{T_g} \\ \hat{T}_w &= \frac{T_w}{T_g}\end{aligned}\quad (4)$$

into equations (1) to (3) gives

$$\frac{D\hat{T}}{Dt} = 2 St_g (\hat{T}_w - \hat{T})\hat{T} \quad (5)$$

$$\frac{\partial \hat{T}_w}{\partial t} = St_w (\hat{T} - \hat{T}_w) \quad (6)$$

$$\frac{\partial \hat{u}}{\partial x} = \frac{1}{\hat{T}} \frac{D\hat{T}}{Dt} \quad (7)$$

where

$$St_g \equiv \frac{hRt_f T_g}{rMpc_p} = \frac{ht_f}{r\rho_g c_p} = \frac{1}{\hat{r}} \left(\frac{h}{\rho_g c_p u_L} \right) \quad (8)$$

$$St_w \equiv \frac{ht_f}{l_w \rho_w c_w} = \frac{1}{\hat{l}_w} \left(\frac{h}{\rho_w c_w u_L} \right) \quad (9)$$

The numbers St_g and St_w have the form of Stanton numbers modified by the presence of the dimensionless lengths \hat{r} and \hat{l}_w , respectively. The use of a parameter St_w , containing both fluid and wall properties, is unusual. The ratio

$$\frac{St_g}{St_w} = \frac{l_w \rho_w c_w}{r \rho_g c_p}$$

which is equal to one-half the ratio of the heat capacity of the wall to the heat capacity of the gas, could be used in place of St_w . However, St_w has been retained since it arises naturally in the development of the equations.

It is seen that St_g and St_w completely determine the differential equations for the dimensionless dependent variables \hat{T} , \hat{T}_w , and \hat{u} . It is shown in reference 2 that the dimensionless constants

$$\hat{L}_O \equiv \frac{L_O}{L_f - L_O} \quad (10)$$

$$\hat{T}_L \equiv \frac{T_L}{T_g} \quad (11)$$

enter the initial and boundary conditions for the dimensionless equations. Within the assumptions made thus far St_g , \hat{L}_O , and \hat{T}_L are constant for a given problem. The St_w will vary only if c_w is allowed to vary.

Pressurant Mass Ratio

Defining an ideal pressurant mass

$$m_i = \pi r^2 (L_f - L_o) \rho_g$$

it can be shown that the mass ratio (sometimes called collapse factor) is given by

$$\frac{m}{m_i} = \int_0^{\hat{L}_f} \frac{d\hat{x}}{\hat{T}(\hat{x}, 1)} - \int_0^{\hat{L}_O} \frac{d\hat{x}}{\hat{T}(\hat{x}, 0)} \quad (12)$$

The mass ratio is, therefore, known when the solution of equations (4), (5), and (6) for the dimensionless temperature variation $\hat{T}(\hat{x}, \hat{t})$ is known. These considerations lead to the following conclusion: With the assumptions stated in the analysis, and with the further assumption that the wall specific heat is constant, the mass ratio is completely determined by the specification of four dimensionless constants St_g , St_w , \hat{L}_O , and \hat{T}_L .

This conclusion is not restricted to any particular liquid, pressurizing gas, or tank wall material. The constant \hat{L}_O is determined by the initial ullage ratio, and the constant \hat{T}_L is determined by the saturation temperature and the pressurizing gas temperature. All other characteristics of the problem, for example tank wall material, wall thickness, tank radius, density, and specific heat of the pressurizing gas and tank pressure, enter only through the constants St_g and St_w . Within the assumptions of the analysis, therefore, a complete parametric investigation can be done by examining the effects on the mass ratio of variations in St_g , St_w , \hat{L}_O , and \hat{T}_L .

For hydrogen problems, however, the assumption that c_w is constant is not very good. If this assumption is dropped, the preceding conclusion no longer holds. The specific heat c_w then varies with temperature T_w and the form of the variation may change from one wall material to another. This leads to the following conclusion: With the assumptions stated in the analysis, and confining attention to a single wall material, the mass ratio is completely determined by the specification of four dimensionless constants St_g , St_w , \hat{L}_O , and \hat{T}_L and the inlet gas temperature T_g .

Effect of Parameters

The parametric investigation is then continued as follows. Values of \hat{L}_O , \hat{T}_L , and T_g are fixed and computer solutions of equations (5), (6), and (7) for a wide range of values of St_g and St_w are obtained. From these solutions (in particular, the temperature distributions) the mass ratios are computed. The results of these calculations are shown in figure 3.

For fixed values $\hat{L}_O = 0.0526$ (corresponding to an initial ullage of 5 percent), $\hat{T}_L = 0.074$, and $T_g = 500^\circ \text{R}$, figure 3 enables the prediction of pressurant mass ratio (collapse factor) for a wide range of design conditions, within the assumptions of the analysis.

The effect of the arbitrarily chosen values of T_g , \hat{T}_L , and \hat{L}_O is examined next. Representative curves ($St_g = 5.0$ and $St_w = 2.5$) are taken from figure 3. With these curves for comparison the value of T_g is changed to 300° and 700°R with \hat{T}_L and \hat{L}_O held at their original values. Again mass ratios are obtained from computer solutions and the results are compared with the original results (fig. 3) for $T_g = 500^\circ \text{R}$. Figure 4 gives an indication of the effect of T_g on the mass ratio. The effect is large only for large values of St_g .

In a similar manner the effect of \hat{T}_L is found by holding T_g and \hat{L}_O fixed at the values used for figure 3 and changing \hat{T}_L to 0.12 ($\hat{T}_L = 0.12$ corresponds to $T_L = 60^\circ \text{R}$ and $\hat{T}_L = 0.074$ corresponds to $T_L = 37^\circ \text{R}$). The results are shown in figure 5. The effect on the mass ratio is small.

The dimensionless initial ullage height \hat{L}_O is treated similarly, changing it from the value 0.0526 (corresponding to an initial ullage volume of 5 percent) used in figure 3 to the value 0.25 (corresponding to an initial ullage volume of 20 percent). As shown in figure 6 the initial ullage effect is small for values of initial ullage up to 20 percent.

Figures 4 to 6 indicate that the reference Stanton number map (fig. 3) has a wider range of validity than was first evident. In particular, the use does not appear to be restricted to the particular values of T_g , \hat{T}_L , and \hat{L}_O that were used to obtain figure 3. This conclusion will be checked against experimental data in a later section.

Effect of Assumptions

It is possible to examine, in a similar manner, the effect of some of the assumptions entering the analysis. Figure 7 shows results obtained using a variable gas specific heat. The difference is negligible. Figure 8 shows the relatively large effect, on the other hand, of choosing wall specific heat to be constant. It was this latter result that led to the inclusion of varying wall specific heat in determining the reference Stanton number map. It is interesting that changing the wall material from stainless steel to aluminum has

little effect on the mass ratio (fig. 9). The reason for this is given in reference 2.

It is shown in reference 2 that the choice of initial values of gas and wall temperatures affects the mass ratio little for initial ullages up to 20 percent. The effects of initial transients in outflow rate and inlet gas temperature are shown in that report to be small. Transient pressure effects are more important.

Comparison with Experiment

An analysis of the tank pressurization problem has indicated that the primary parameters affecting the mass required to pressurize a cylindrical tank during outflow can be combined into two dimensionless groups having the form of modified Stanton numbers, one associated with the gas and one with the tank wall. This enables approximate values of mass ratio (collapse factor) to be determined from a single figure for a large range of design variables. To test this conclusion the experimental data used previously in the paper will be used again.

In the case of the Lewis experiments and the Lockheed-Georgia experiments described before, the experimental average values of heat-transfer coefficient are available. Using these values of h the gas and wall Stanton numbers can be determined for each set of data. Using these Stanton numbers and figure 3, an estimated value of mass ratio can be obtained.

Values of mass ratio determined in this way for the Lewis experiments are shown in table IV. One of the Lockheed-Georgia experiments was omitted since it contained helium in the initial ullage space and was subsequently pressurized

TABLE IV. - COMPARISON OF EXPERIMENTAL VALUES OF MASS RATIO WITH VALUES DETERMINED FROM THE REFERENCE STANTON NUMBER MAP

Example	Experi- mental mass ratio	Mass ratio determined from Stanton number map	Percent differ- ence	Example	Experi- mental mass ratio	Mass ratio determined from Stanton number map	Percent differ- ence
Lewis data				Lockheed-Georgia data			
1	2.58	2.76	-6.5	1	1.72	1.54	11.7
2	1.77	1.84	-3.8	2	2.14	2.16	-0.9
3	3.09	3.31	-6.7	3	1.79	1.61	11.2
4	3.20	2.25	-2.2	4	1.71	1.68	1.8
5	1.47	1.35	8.9	5	1.81	1.69	7.1
6	2.37	2.56	-7.4	7	1.80	1.71	5.3
7	2.73	2.86	-4.5	8	1.75	1.59	10.0
8	1.86	1.93	-3.6	9	1.83	1.69	8.3
9	1.38	1.25	10.4				
10	3.92	4.25	-7.8				

with hydrogen, a situation not covered by figure 3. Values of mass ratio determined from figure 3 for the other eight Lockheed-Georgia experiments are shown in table IV. Also shown are the experimental values of mass ratio for all the experiments. In the case of the Lewis data the actual experimental values are modified to eliminate the heat sink effect that is not accounted for in figure 3.

The percent difference between calculated and experimental values of mass ratio is also shown in table IV. The average value of the absolute differences for the Lewis data is about 6 percent. For the Lockheed-Georgia data the average is about 7 percent. These results bear out the implications of the parametric analysis.

It should be remembered, however, that to compute Stanton numbers for design purposes a value of heat-transfer coefficient h must be estimated. In reference 2 a simple method of estimating h from a free convection formula is examined. For the experiments considered here such a simple method appears to be adequate. Its general use, however, is open to serious question and the determination of heat-transfer coefficient for arbitrary conditions remains an unsettled question.

APPENDIX - SYMBOLS

C	effective perimeter of internal hardware
c_p	specific heat of gas
c_w	specific heat of tank wall
h	heat-transfer coefficient
J	mechanical equivalent of heat
L_f	ullage height at time $t = t_f$
\hat{L}_f	dimensionless ullage height, $L_f/(L_f - L_0)$
L_0	ullage height at time $t = 0$
\hat{L}_0	dimensionless initial ullage height, $L_0/(L_f - L_0)$
l	height of ullage (see fig. 1)
l_w	thickness of tank wall
\hat{l}_w	dimensionless thickness of tank wall, $l_w/(L_f - L_0)$
M	molecular weight

m	mass of pressurant gas added during outflow
m_i	mass of pressurant gas required assuming no heat transfer
P	pressure in tank
q_I	heat flow rate to gas from internal hardware
q_O	heat flow rate to tank wall from outside
R	universal gas constant
r	radius of tank
\hat{r}	dimensionless radius of tank, $r/(L_f - L_o)$
St_g	modified gas Stanton number, $\frac{1}{\hat{r}} \frac{h}{\rho_g c_p u_L}$
St_w	modified wall Stanton number, $\frac{1}{\hat{r}_w} \frac{h}{\rho_w c_w u_L}$
T	gas temperature
\hat{T}	dimensionless gas temperature, T/T_g
T_g	gas temperature at tank inlet
T_L	gas temperature at liquid interface
\hat{T}_L	dimensionless temperature, T_L/T_g
T_w	temperature of tank wall
\hat{T}_w	dimensionless temperature, T_w/T_g
t	time
\hat{t}	dimensionless time, t/t_f
t_f	time at end of outflow
Δt	time increment for finite difference equations
u	velocity of gas
\hat{u}	dimensionless gas velocity, u/u_L
u_L	velocity of gas at liquid interface
x	space coordinate in direction of tank axis
\hat{x}	dimensionless space coordinate, $x/(L_f - L_o)$

Δx space increment for finite difference solution
 Z compressibility factor
 ρ_g density of gas
 ρ_w density of tank wall

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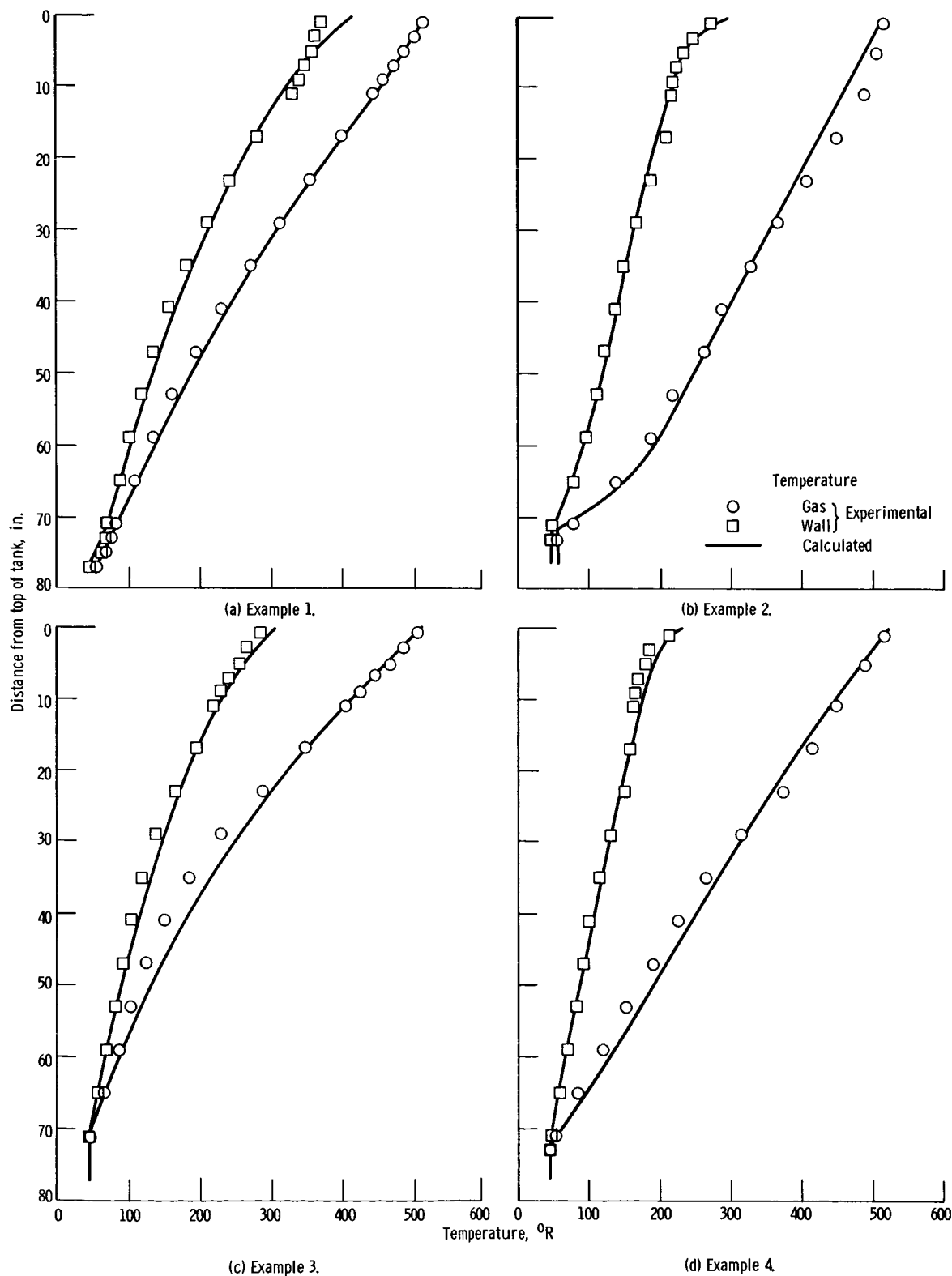


Figure 2. - Comparison of calculated and experimental gas and wall temperatures at end of outflow.

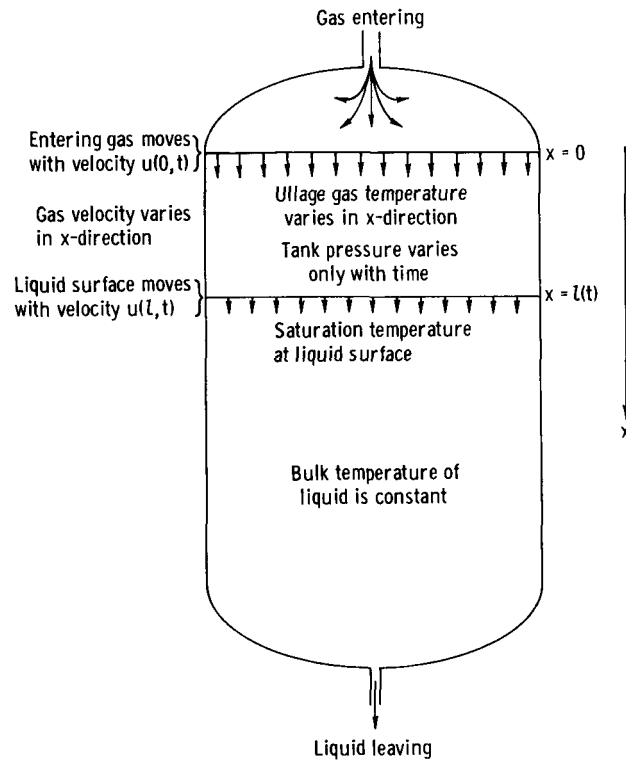


Figure 1. - Schematic drawing of cylindrical tank.

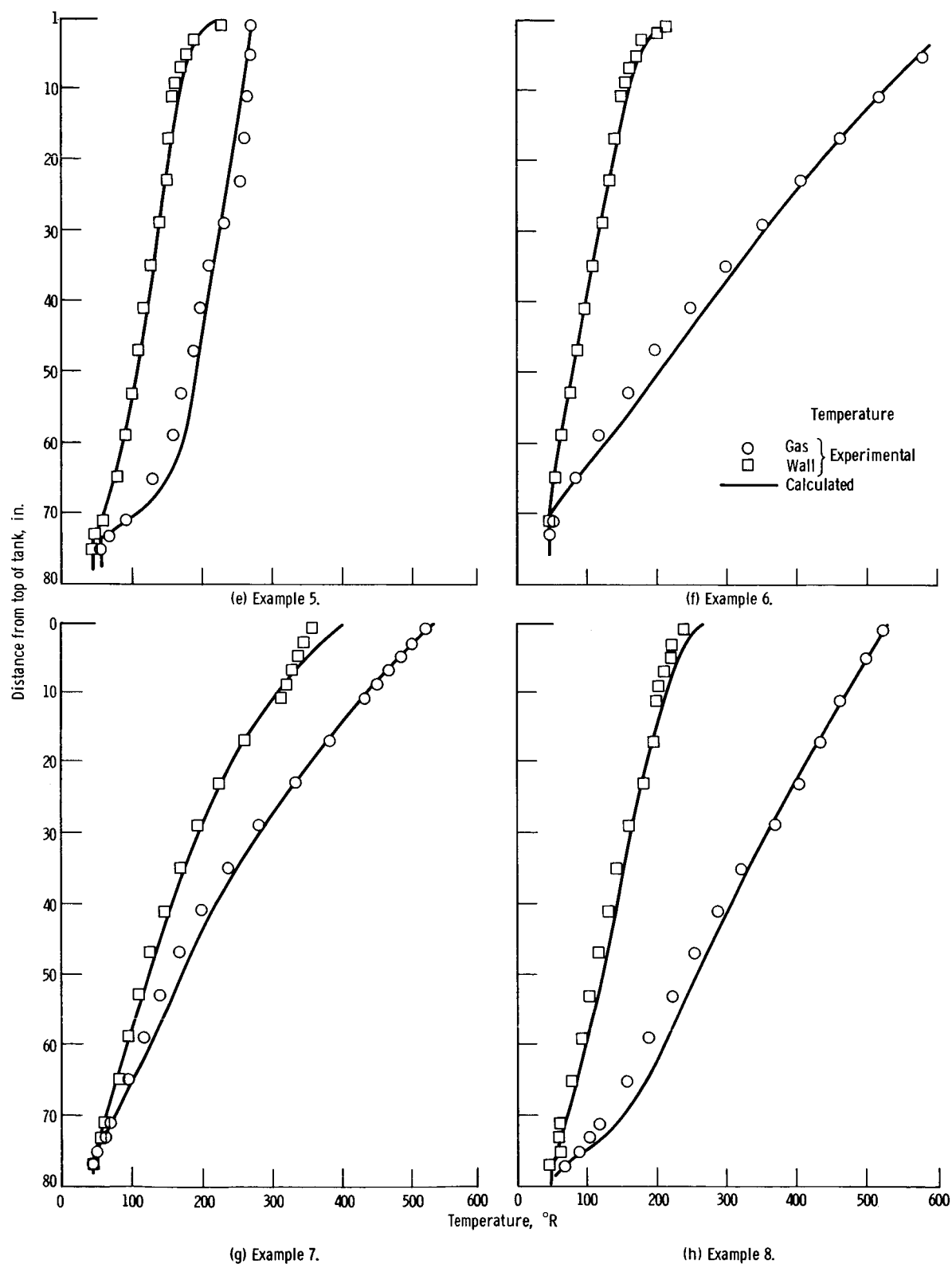
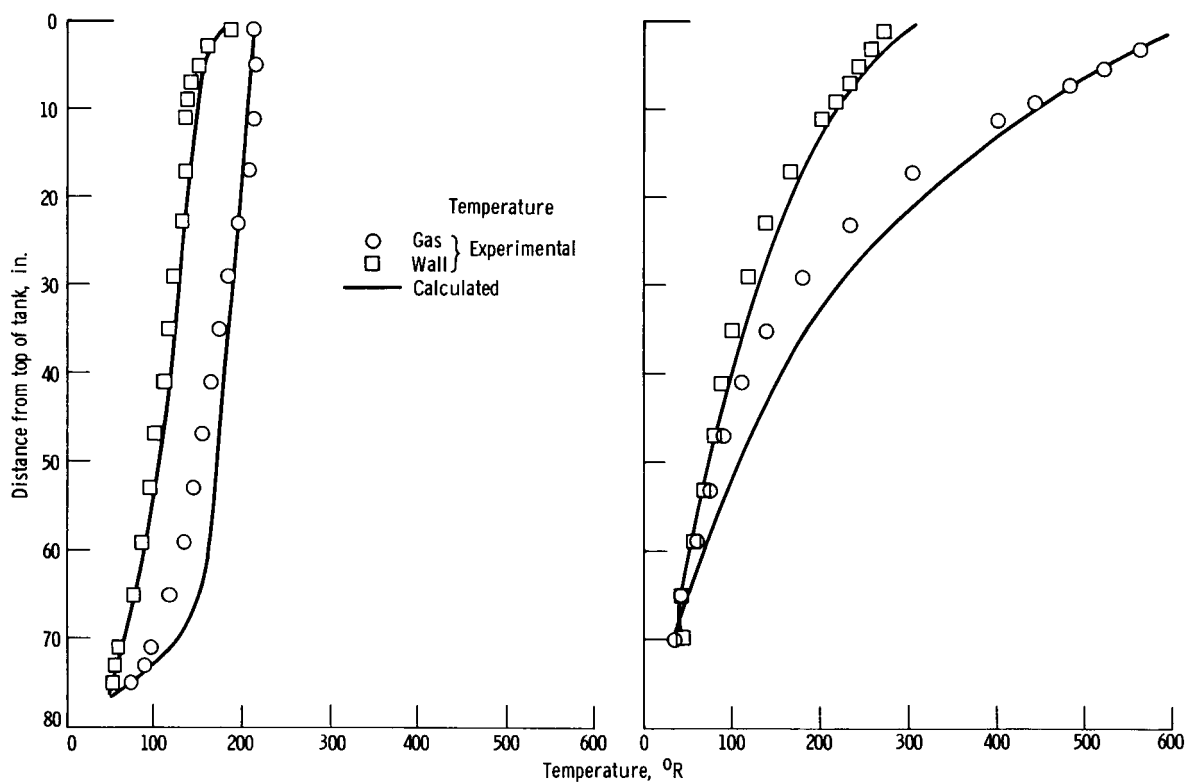


Figure 2. - Continued. Comparison of calculated and experimental gas and wall temperatures at end of outflow.



(i) Example 9.

(j) Example 10.

Figure 2. - Concluded. Comparison of calculated and experimental gas and wall temperatures at end of outflow.

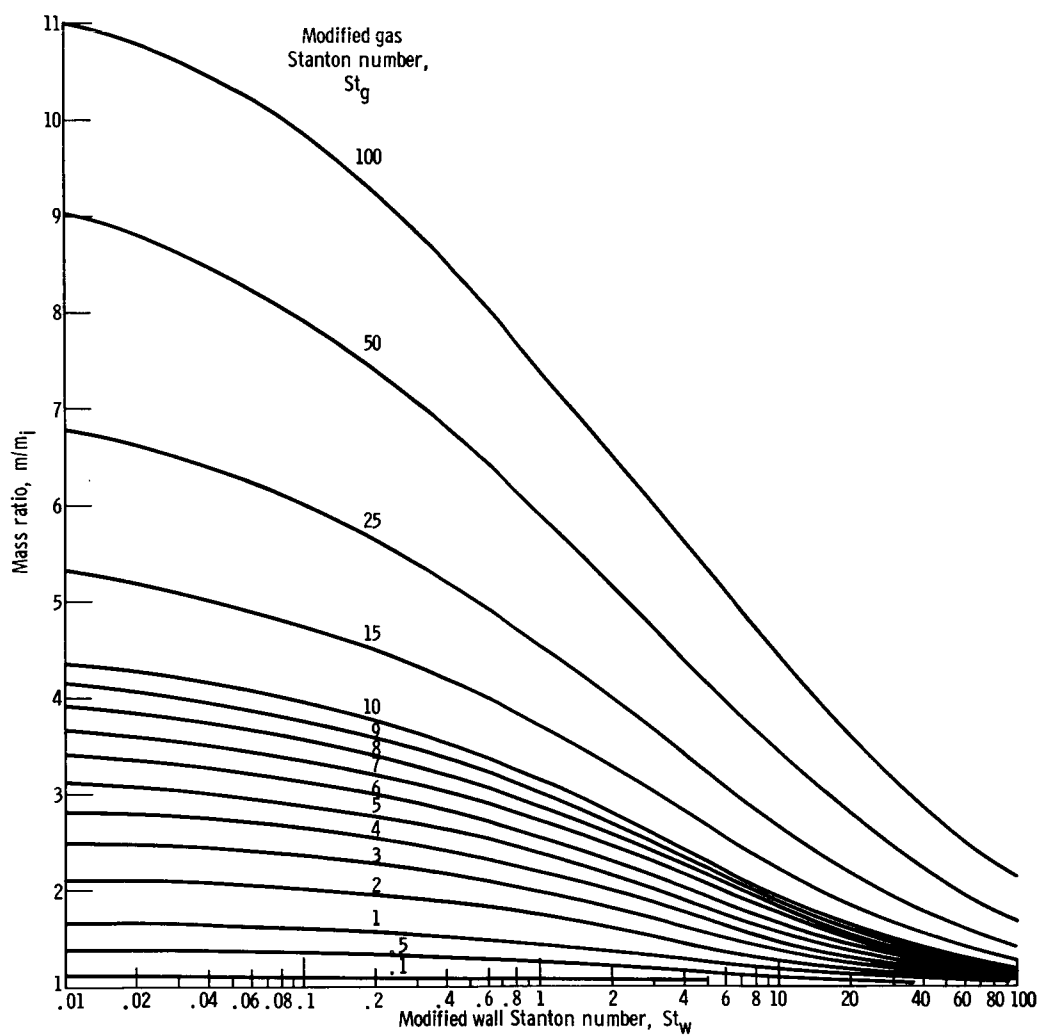


Figure 3. - Stanton number map showing values of mass ratio for range of gas and wall Stanton numbers. Initial ullage ratio, 0.05; dimensionless interface temperature, 0.074.

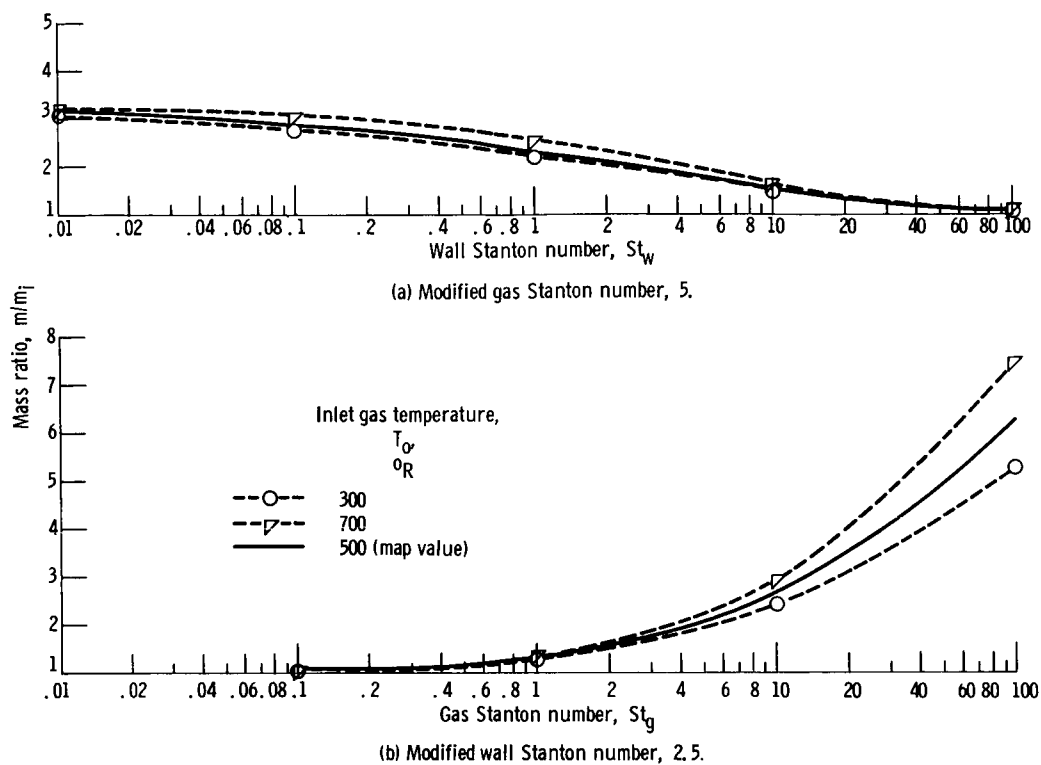


Figure 4 - Effect on mass ratio of changing the inlet gas temperature. Dimensionless interface temperature, 0.074; dimensionless initial ullage height, 0.0526.

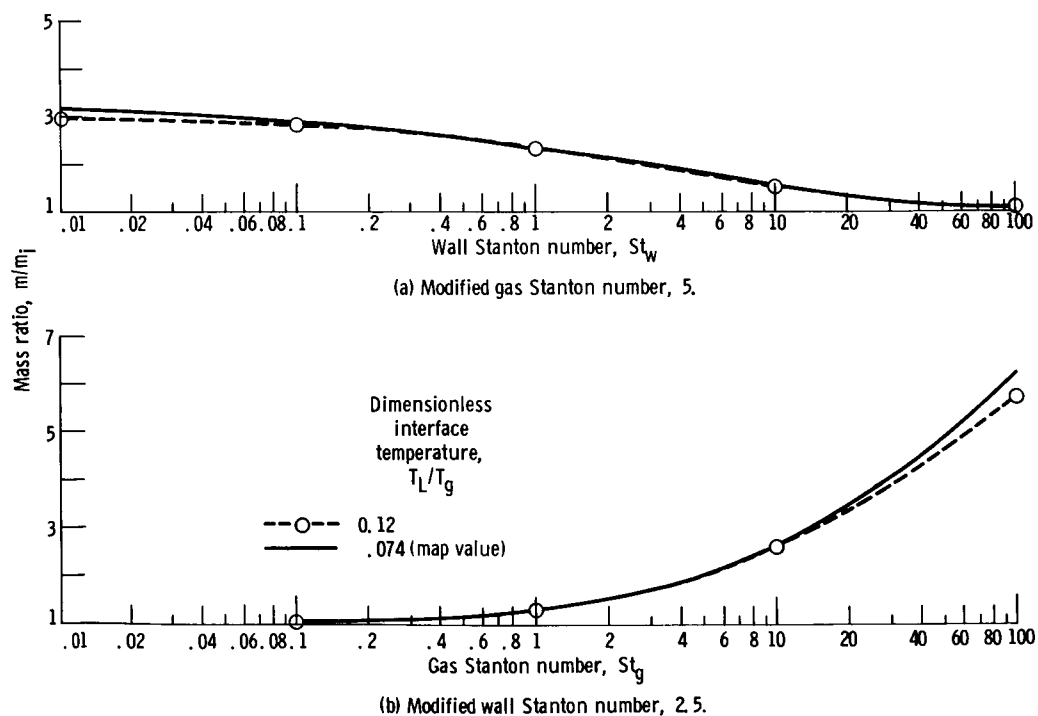


Figure 5 - Effect on mass ratio of changing the dimensionless interface temperature. Inlet gas temperature, $500^{\circ}R$; dimensionless initial ullage height, 0.0526.

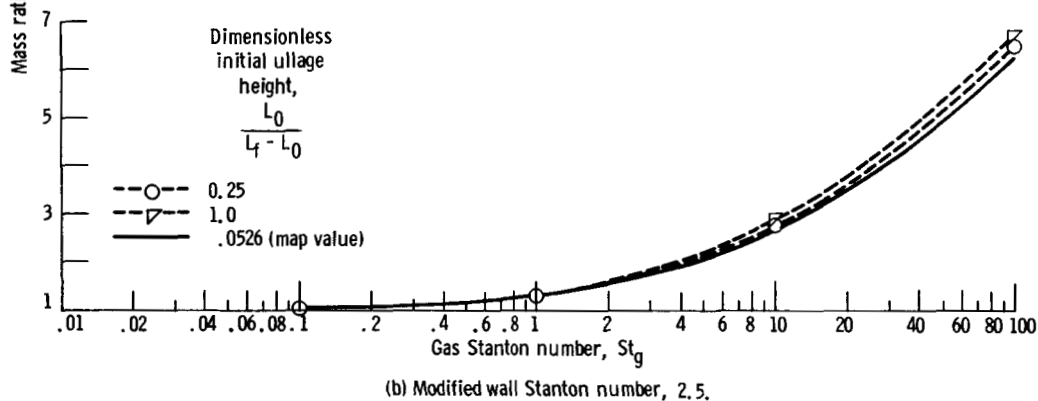
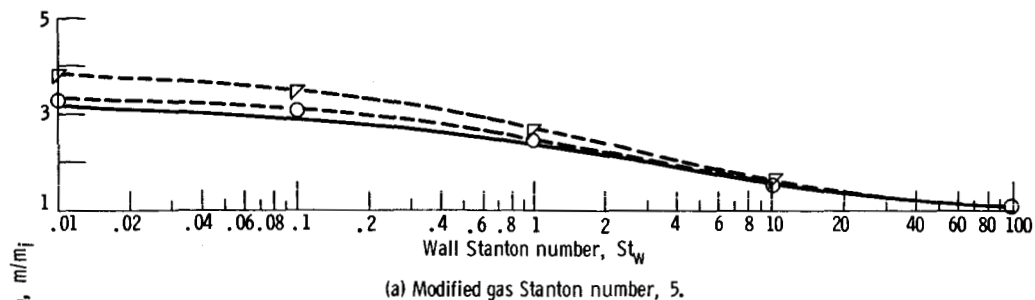


Figure 6. - Effect on mass ratio of changing the dimensionless initial ullage height. Inlet gas temperature, 500°R ; dimensionless interface temperature, 0.074.

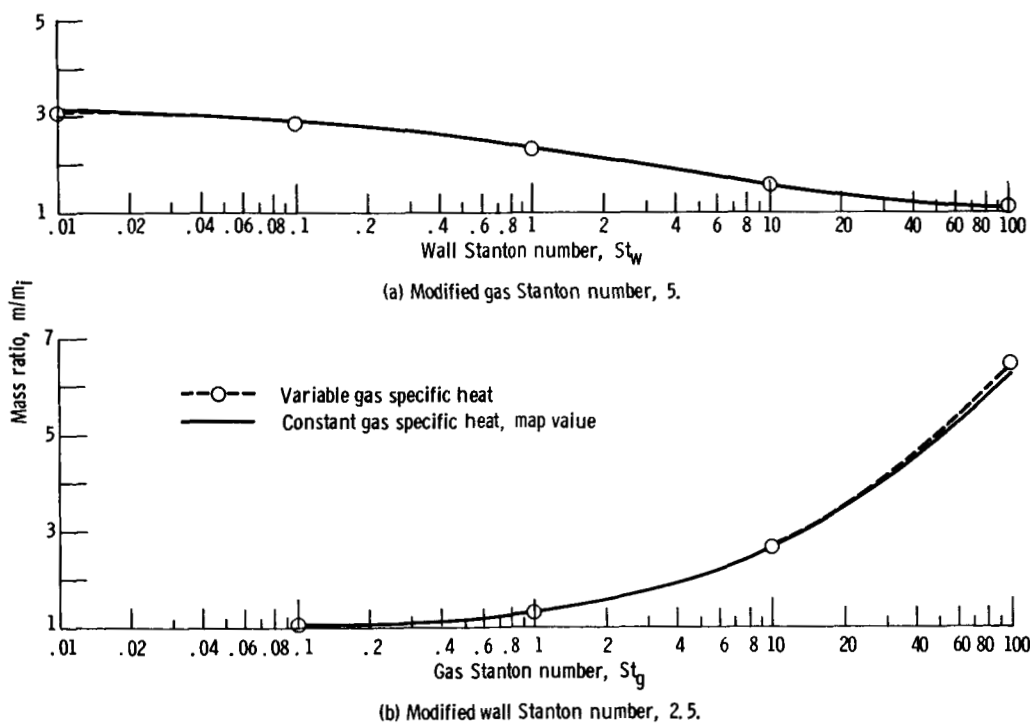
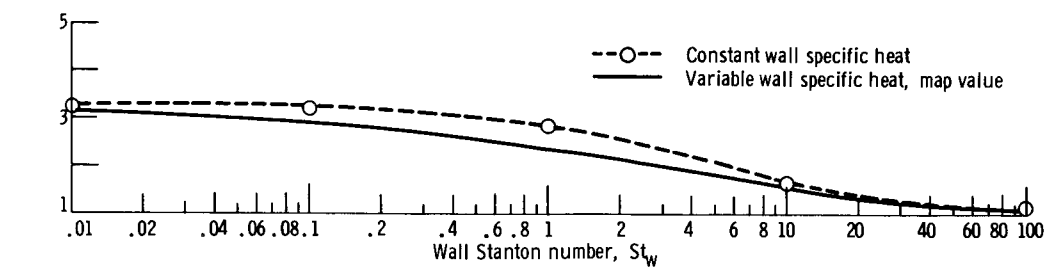
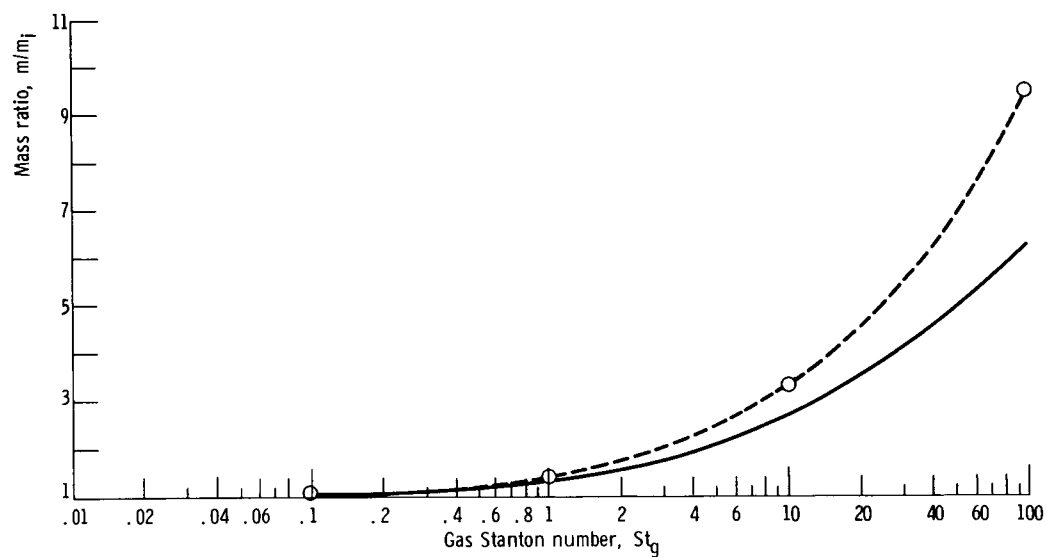


Figure 7. - Effect on mass ratio of allowing gas specific heat to vary with temperature. Inlet gas temperature, 500°R ; dimensionless interface temperature, 0.074; dimensionless initial ullage height, 0.0526.

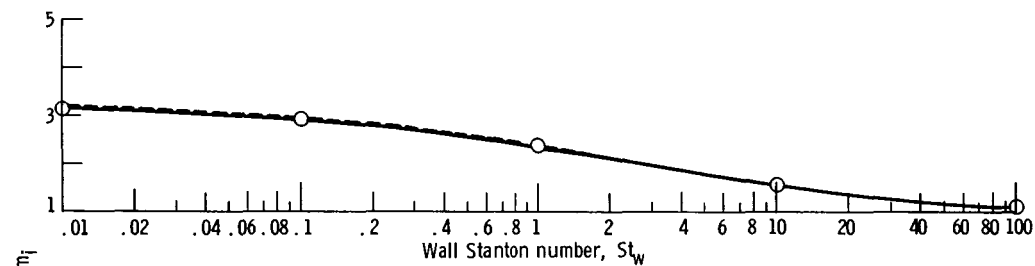


(a) Modified gas Stanton number, 5.

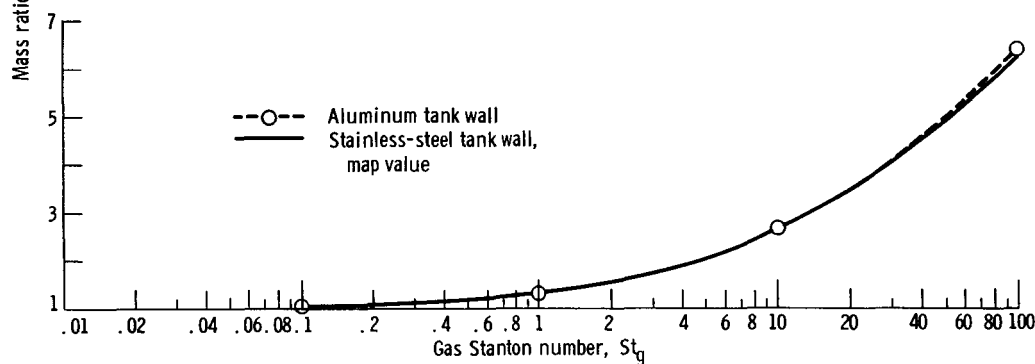


(b) Modified wall Stanton number, 2.5.

Figure 8. - Effect on mass ratio of holding wall specific heat constant. Inlet gas temperature, 500° R; dimensionless interface temperature, 0.074; dimensionless initial ullage height, 0.0526.



(a) Modified gas Stanton number, 5.



(b) Modified wall Stanton number, 2.5.

Figure 9. - Effect on mass ratio of changing tank wall temperature from stainless steel to aluminum. Dimensionless interface temperature, 0.074; dimensionless initial ullage height, 0.0526.